

Math 103A Fall 2020

Homework 1

Name: \_\_\_\_\_

Section: \_\_\_\_\_

**For all problems, you must elaborate your work mathematically**

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1. [1pt] Show that for  $a, b, c \in \mathbb{Z}$

(a) If  $a + b = a + c$ , then  $b = c$

(b) If  $ab = ac$ , then  $b = c$ .

2. [1pt] Use the Euclidean algorithm to compute  $d = \gcd(456, 1234)$ . Find  $m, n \in \mathbb{Z}$  such that  $d = 456m + 1234n$ .

3. [1pt] Show that an interval  $[13, 17] = \{t \in \mathbb{R} \mid 13 \leq t \leq 17\}$  is not well-ordered.

4. [1pt] Show that there is no finite ordered integral domain.

5. [1pt] Show that for  $x \in \mathbb{Z}$ , if  $6 \mid x^2$  then  $6 \mid x$ .

optional Describe which integer  $n$  satisfies " $n \mid x^2$  implies  $n \mid x$ ". (In other words, what integers satisfy this statement?)