

Math 103A Fall 2020

Quiz 1

(Time Limit : 2 hours, Submit it by Wednesday (10/21) at PDT 8:20pm)

Name: _____

Section: _____

1. [20pts] Prove the following using integer axioms :

(a) [6pts] For $c < 0$, prove that $c^3 < 0$.

(b) [6pts] Prove that there is no positive integer smaller than 1.

(c) [8pts] Remind that $n \in \mathbb{Z}$ is *even* if $n \equiv 0 \pmod{2}$ and *odd* if $n \equiv 1 \pmod{2}$. For $n \in \mathbb{Z}$, prove that n is either even or odd **without using division algorithm**.

(**Hint** : let K be an integer that is neither even nor odd, define

$S = \{n \in \mathbb{Z}_+ \mid n \text{ is even or odd and } n \geq K\}$ and use the problem **1(b)**)

2. [20pts] Find a positive solution $x \in \mathbb{Z}_+$ satisfying that

(a) [10pts] $2x + 21 \equiv 16 \pmod{45}$.

(b) [10pts] $49x^2 - 4 \equiv 0 \pmod{11}$.

3. [15pts]

(a) [7pts] Using Euclidean algorithm, find $d = \gcd(180, 239)$.

(b) [8pts] Find $n, m \in \mathbb{Z}$ such that $d = 180n + 239m$.

4. [15pts]

(a) [5pts] Let $p > 2$ be a prime. Explain why either $p \equiv 1 \pmod{4}$ or $p \equiv 3 \pmod{4}$.

- (b) [5pts] List all primes $p < 20$ satisfying $p \equiv 3 \pmod{4}$.
- (c) [5pts] Using (a) and (b), prove that there is a prime of the form $4n - 1$ bigger than 20.

5. [30pts] Which of the following is a group? Justify your answer.

- (a) [10pts] (\mathbb{Z}, \times) (the set of integers with multiplication)
- (b) [10pts] $A = \{180m + 239n \mid \text{for } m, n \in \mathbb{Z}\}$ with addition $+$.
- (c) [10pts] (\mathbb{Z}, \star) where $a \star b = ab - a - b + 1$