

**Math 20D**  
**Fall 2022**  
**Midterm 1**  
**10/17/22**  
**Instructor : Kisun Lee**  
**Time Limit: 50 Minutes**

**Name (Print) :** \_\_\_\_\_  
**Section :** \_\_\_\_\_

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This exam contains 6 pages (including this cover page) and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **If you use a “fundamental theorem” you must indicate this** and explain why the theorem may be applied.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit.** A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Problem	Points	Score
1	2	
2	4	
3	8	
4	16	
5	10	
Total:	40	

Do not write in the table to the right.

1. (2 points) True/False question.

(a) (1 point) Let  $F_1(x, y, \frac{dy}{dx}, \dots, \frac{d^n y}{dx^n}) = 0$  and  $F_2(x, y, \frac{dy}{dx}, \dots, \frac{d^n y}{dx^n}) = 0$  be linear differential equations. Then, in general,  $F_1^2 - 3F_2 = 0$  is also linear. (T/F)

(b) (1 point) It is ensured that the initial value problem

$$y^6 \frac{dy}{dx} = x^6, \quad y(5) = 0$$

has the unique solution. (T/F)

2. (4 points) Show that  $y^8 + x - 1 = 0$  is an implicit solution to

$$\frac{dy}{dx} = -\frac{1}{8y^7}.$$

3. (8 points) Solve the following differential equations. Solve for  $y$  if possible.

$$3t + y + 6 - \frac{dy}{dt} = 0$$

4. (16 points) Solve the following initial value problems.

(a) (8 points)

$$\sqrt{y}dx + (x - 5)dy = 0, \quad y(6) = 16$$

(b) (8 points)

$$y'' + 25y = 0; \quad y(0) = 1, \quad y'(0) = 3.$$

5. (10 points) Find an integrating factor of the form  $x^n y^m$  and solve the equation :

$$(3y^2 - 6x^{-1})dx + (4xy - 9y^{-1})dy = 0$$