Math 1711   **Name (Print):** _____
Fall 2019
Midterm 1
9/12/19
Time Limit: 75 Minutes   Section _____

This exam contains 7 pages (including this cover page) and 10 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **If you use a "fundamental theorem" you must indicate this** and explain why the theorem may be applied.

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.

- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.

- If you need more space, use the back of the pages; clearly indicate when you have done this.

Do not write in the table to the right.

Formula $- \; n(R \cup S \cup T) = n(R) + n(S) + n(T) - n(R \cap S) - n(R \cap T) - n(S \cap T) + n(R \cap S \cap T)$

$- \; P(n, r) = \frac{n!}{(n-r)!} = n \cdot (n-1) \cdots (n - r + 1)$

$- \; C(n, r) = \frac{P(n,r)}{r!}$

$- \; \binom{n}{n_1, n_2, \cdots, n_m} = \frac{n!}{n_1! n_2! \cdots n_m!}$

Problem	Points	Score
1	10	
2	12	
3	10	
4	10	
5	12	
6	12	
7	10	
8	12	
9	12	
10	0	
Total:	100	

1. (10 points) One hundred and sixty business executives were surveyed to determine whether they regularly visit the *CNN Money, Bloomberg*, or *The Wall Street Journal* websites. The survey showed that 70 visit *CNN*, 60 visit *Bloomberg*,, 55 visit *The Wall street Journal*, 45 visit exactlly two of the three websites, 20 visit *CNN* and *Bloomberg*, 20 visit *Bloomberg* and *The Wall Street Journal*, and 5 visit all three websites.

   (a) (4 points) Draw a Venn diagram displaying the characteristics of the students.

   (b) (3 points) How many visit only *CNN*?

   (c) (3 points) How many do not visit any of the three websites?

2. (12 points) Let R, S and T be subsets of U with the following data.

$$n(R') = 23, n(R \cup S) = 21, n(S) = 14, n(T) = 22,$$
$$n(R \cap S) = 7, n(S \cap T) = 9, n(R \cap T) = 11, n(R \cap S \cap T) = 5$$

(a) (6 points) Draw an appropriate Venn diagram, and use the given data to determine the number of elements in each basic region.

(b) (3 points) Shade the following region on the Venn diagram.

$$(R \cap T) \cup (S \cap T')$$

(c) (3 points) Find the number of elements in

$$(R \cap T) \cup (S \cap T')$$

3. (10 points) If $P(6, r) = 360$ and $C(6, r) = 15$, find r.

4. (10 points) In a higher secondary exam, 80% of the students have passed in Literature and 85% in Mathematics, while 75% passed in both. If 35 students failed in both, find the total number of students.

5. (12 points) Determine the coefficient of $x^4 y^3$ in the expansion of $(x - 4y)^7$. Simplify your answer as far as possible.

6. (12 points) Calculate the value for

$$C(11, 0) - C(11, 1) + C(11, 2) - C(11, 3) + \cdots + C(11, 10) - C(11, 11)$$

using binomial theorem.

7. (10 points) A hot dog stand has possible toppings of mustard, ketchup, chili, pickles and sauerkraut.

   (a) (5 points) How many possible hot dogs can be ordered?

   (b) (5 points) How many hot dogs have at least one topping?

8. (12 points) A standard deck of cards contains 52 cards. A eight card hand is drawn from the deck. In how many ways can the hand contain a pair and two triples of different denominations? You do not need to simplify your final answer.

9. (12 points) Solve the following problems. You don't need to show your work. You don't need to simplify your final answer.

   (a) (2 points) Find the number of possible 4-digit locker codes. **(no partial credit)**

   (b) (2 points) There are 10 students. How many ways to elect a president, VP and secretary? **(no partial credit)**

   (c) (2 points) To complete a test, you must work three of the first five problems, then two of the next three problems, and four of the last six problems. In how many different ways can the test be completed? **(no partial credit)**

   (d) (2 points) Evaluate $C(8, 6)$. **(no partial credit)**

   (e) (2 points) A group of 30 students is divided among three dormitories: 10 students in dorm A, 12 in dorm B, and 8 in dorm C. In how many ways can the students be distributed among the three dorms? **(no partial credit)**

   (f) (2 points) In how many ways can a group of 24 children be divided into 6 groups of 4 children? You may assume all groups are identical. **(no partial credit)**

10. (5 points)

(Bonus Problem) Determine the coefficient of $x^4 y^6$ in the expansion of $(x - y^2)^7$. Simplify your answer as far as possible.