Math 1555
Name (Print):
Spring 2018
Midterm 2
3/1/18
Time Limit: 50 Minutes
Section

This exam contains 7 pages (including this cover page and the optional problem) and 6 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may not use your books, notes, or any calculator on this exam.
You are required to show your work on each problem on this exam. The following rules apply:

- If you use a "fundamental theorem" you must indicate this and explain why the theorem may be applied.
- Organize your work, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- Mysterious or unsupported answers will not receive full credit. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

| Problem | Points | Score |
| :---: | :---: | :---: |
| 1 | 15 |  |
| 2 | 20 |  |
| 3 | 15 |  |
| 4 | 20 |  |
| 5 | 15 |  |
| 6 | 15 |  |
| Total: | 100 |  |

- Use pen to write your answers. If you use a pencil then, you will lose your right to regrade your exam.

Do not write in the table to the right.

1. (15 points) (a) (5 points) Show that

$$
\frac{1}{\sqrt{1+x^{2}}} \geq \frac{1}{2 x}>0
$$

for $x \geq 1$.
(b) (10 points) Use your result in (a) to show that

$$
\int_{1}^{\infty} \frac{1}{\sqrt{1+x^{2}}} d x
$$

is divergent (It is okay to use the result in (a) without solving (a)).
2. (20 points) Evaluate integral by following steps below:

$$
\int \frac{1}{x(x+1)^{2}} d x
$$

(a) (10 points) Use partial-fraction decomposition to $\frac{1}{x(x+1)^{2}}$.
(b) (10 points) Evaluate $\int \frac{1}{x(x+1)^{2}} d x$ using the result of (a).
3. (15 points) Evaluate the integral using integration by parts.
(a) (5 points)

$$
\int x \sin (1-x) d x
$$

(b) (10 points)

$$
\int e^{-x} \cos (2 x) d x
$$

4. (20 points) Evaluate integral by following steps below:

$$
\int \frac{x^{3}+4 x+1}{x^{2}+4} d x
$$

(a) (10 points) Use long division in order to make $\frac{x^{3}+4 x^{2}+1}{x^{2}+4}$ proper (the degree of numerator is less than the degree of denominator).
(b) (10 points) Use trigonometric substitution and the result of (a) to evaluate $\int \frac{x^{3}+4 x^{2}+1}{x^{2}+4} d x$. (Hint : $1+\tan ^{2} \theta=\sec ^{2} \theta$ )
5. (15 points) Evaluate the integral by making the substitution.
(a) (5 points)

$$
\int_{0}^{1} x e^{1-2 x^{2}} d x
$$

(b) (10 points)

$$
\int 3 x^{5} \sqrt{4+x^{3}} d x
$$

6. (15 points) Compute the Taylor polynomial $P(x)$ of degree 2 about 1 for the following function $f(x)$

$$
f(x)=x^{1 / 5}
$$

Optional (5 points) Approximate $f(0.9)$ using $P(x)$ above.

