Math 1552
Summer 2019
Midterm 1
6/13/19
Time Limit: 75 Minutes
Section $\qquad$

This exam contains 9 pages (including this cover page) and 6 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may not use your books, notes, or any calculator on this exam.
You are required to show your work on each problem on this exam. The following rules apply:

- If you use a "fundamental theorem" you must indicate this and explain why the theorem may be applied.
- Organize your work, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- Mysterious or unsupported answers will not receive full credit. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages;

| Problem | Points | Score |
| :---: | :---: | :---: |
| 1 | 10 |  |
| 2 | 50 |  |
| 3 | 10 |  |
| 4 | 10 |  |
| 5 | 20 |  |
| 6 | 0 |  |
| Total: | 100 |  | clearly indicate when you have done this.

Do not write in the table to the right.

## Formulae

$$
\begin{gathered}
\sin ^{2} x+\cos ^{2} x=1 \\
1+\tan ^{2} x=\sec ^{2} x \\
\cos ^{2} x=\frac{1}{2}(1+\cos 2 x) \\
\sin ^{2} x=\frac{1}{2}(1-\cos 2 x)
\end{gathered}
$$



Figure 1: Unit circle for points $(\cos x, \sin x)$

1. (10 points) (a) (5 points) Draw the graph of $y=\sin x$ and $y=\cos x$ over $[0, \pi]$ and find the coordinates of intersections.
(b) (5 points) Find the area of the region bounded by $y=\sin x$ and $y=\cos x$ on $[0, \pi]$.
2. (50 points) Evaluate the indefinite integrals.
(a) (10 points)

$$
\int \cos (\ln x) d x
$$

(b) (10 points)

$$
\int \cos ^{4} x d x
$$

(c) (10 points)

$$
\int x^{2}\left(\frac{x^{3}}{18}-1\right)^{5} d x
$$

(d) (10 points)

$$
\int \frac{1}{x^{4} \sqrt{x^{2}-1}} d x
$$

(e) (10 points)

$$
\int \frac{1}{x\left(x^{2}+1\right)} d x
$$

3. (10 points) Evaluate $\int_{-2}^{1}|x+1| d x$
4. (10 points) Find $F^{\prime}(3)$ where

$$
F(x)=\int_{\frac{\pi}{6}}^{4 x} \cos ^{2}\left(\frac{\pi t}{6}\right) d t
$$

5. (20 points) Find $\int_{0}^{1}-x^{2}+x d x$ using $x_{i}^{\star}$ the right-hand endpoint for each subinterval with the following procedure:
(a) (2 points) Find $x_{0}$. (no partial credit)
(b) (2 points) Find $x_{n}$. (no partial credit)
(c) (2 points) Find $\Delta x$. (no partial credit)
(d) (4 points) Find the general form of $x_{i}^{\star}$.
(e) (10 points) Write the Riemann sums $\lim _{n \rightarrow \infty} \sum_{i=1}^{n} f\left(x_{i}^{\star}\right) \Delta x$, and take the limit to find the value of definite integral. (Hint: $\left.\sum_{i=1}^{n} i=\frac{n(n+1)}{2}, \sum_{i=1}^{n} i^{2}=\frac{n(n+1)(2 n+1)}{6}\right)$
6. (5 points) (bonus) Suppose $f(1)=2, f(4)=7, f^{\prime}(1)=5, f^{\prime}(4)=3$ and $f^{\prime \prime}(x)$ is integrable. Evaluate

$$
\int_{1}^{4} x f^{\prime \prime}(x) d x
$$

